Abstracts
Wasserstein calculus and related topics
1st Moscow-UK workshop on stochastic analysis

Pierre Cardaliaguet, Université Paris-Dauphine
The master equation for mean field games with a major agent
Mean field games with a major agent study optimal control problems with infinitely many small controllers facing a major controller. The "value function" of the agents then satisfy a nonlinear nonlocal system of partial differential equations stated in the space of measures. In this joint work with Marco Cirant (U. Padova) and A. Porretta (U. Rome Tor Vergata) we explain how to build short time a classical solution for this system and use the solution to prove the mean field limit of the associated N player game as the number N of the players tends to infinity.

Nicolas Champagnat, Université de Lorraine
Lyapunov criteria for the convergence of conditional distributions of absorbed Markov processes
For Markov processes with absorption, we provide general criteria ensuring the existence and the exponential non-uniform convergence in total variation norm to a quasi-stationary distribution. We also characterize a subset of its domain of attraction by an integrability condition and prove the existence of a right eigenvector for the semigroup of the process. We apply these results to diffusion processes absorbed at the boundary of a domain, to perturbed dynamical systems and discrete time processes evolving in discrete state spaces and improve in all these cases the best known results. This is joint work with Denis Villemonais.

Paul-Eric Chaudru de Raynal, Université Savoie Mont Blanc
Mean reflected Stochastic Differential Equations
In this talk, we will consider (forward or backward) Stochastic Differential Equations (SDE) whose law are constrained to live in an a priori given set. Such systems can be seen as SDE with non-linear (in the McKean-Vlasov sense) reflexion. Well-posedness and particles approximation issues as well as relations with partial differential equations will be discussed.

Dan Crisan, Imperial College London
Smoothing properties of McKean-Vlasov Stochastic Differential Equations
I will present some gradient bounds for solutions of Stochastic Differential Equations with McKean-Vlasov nonlinearity. They are obtained by using integration by parts formulae on Wiener space for solutions of SDEs with general McKean–Vlasov interaction and uniformly elliptic coefficients. These integration by parts formulae hold both for derivatives with respect to a real variable and derivatives with respect to a measure understood in the sense of Lions.

This is joint work with Eamon McMurray (JP Morgan).

Alexander Davie, University of Edinburgh

Multivariate central limit bounds in Vaserstein metrics.

We give some bounds in Vaserstein W_p metrics for central limit and related Gaussian approximations for vector-valued random variables. We also describe asymptotic expansions for W_2 distances between approximately Gaussian distributions, with applications to monotonicity questions.

Franco Flandoli, Scuola Normale Superiore of Pisa

Particle system approximation of Vlasov-Navier-Stokes equations

The system of PDEs called Vlasov-Navier-Stokes equations (more precisely Vlasov-Fokker-Planck-Navier-Stokes equation) describes particles in a fluid, exchanging momentum by the so called Stokes drag force, with the particles modeled by a density that satisfies a Vlasov-Fokker-Planck equation. Its origin as a macroscopic limit of true particles embedded into a fluid is still not so clear. The talk will present a recent result in this direction, by Cristiano Ricci, Marta Leocata and the author.

Megan Griffin-Pickering University of Cambridge

A Particle Approximation for the Kinetic Isothermal Euler Equation.

In this talk, I will consider a kinetic equation that models plasma - the Kinetic Isothermal Euler equation. This is the formal limit of the well-known Vlasov-Poisson equation in the quasineutral limit where the Debye length tends to zero.The Vlasov-Poisson equation can itself be derived as the formal limit of a system of finitely many interacting particles, as the number of particles tends to infinity. The rigorous justification of this (mean field) limit remains a major open problem. In recent years several authors have derived the Vlasov-Poisson equation rigorously from various regularised microscopic systems.

I will present a recent joint work with Mikaela Iacobelli, in which we give a rigorous derivation of the Kinetic Isothermal Euler equation from a regularised particle system, using a combined mean field and quasineutral limit. Our approach relies on Wasserstein stability estimates for the Vlasov-Poisson equation.

Alexander Gushchin, Steklov Mathematical Institute

The joint distributions of an increasing process and its compensator

We study the set of probability distributions on the positive quadrant of the plane that are characterised as all possible joint distributions of a locally integrable increasing process and its compensator at some stopping time. We provide a full characterisation of this set under an additional integrability-type assumption. However, this assumption is not necessary satisfied for measures in this set. Much attention is paid to the question how we can reduce or extend the class of locally integrable increasing processes while preserving the set of joint distributions. The corresponding sets of possible pairs of marginals are described. We also present many open problems.

William Hammersley University of Edinburgh

McKean-Vlasov SDEs under Measure Dependent Lyapunov Conditions

The classical localisation procedure used in establishing the existence of solutions to stochastic differential equations in a non-Lipschitzean setting does not directly extend to the McKean-Vlasov SDEs. I will describe how to adapt this approach to prove the existence of weak solutions in the case of continuous coefficients whose growth is controlled by a Lyapunov function. Conditions ensuring the uniqueness of solutions will also be discussed.

Elena Issoglio, University of Leeds

A non-linear parabolic PDE with a distributional coefficient and its applications to stochastic analysis

We consider a non-linear parabolic partial differential equation (PDE) on R^d with a distributional coefficient in the non-linear term. The distribution is an element of a Besov space with negative regularity and the non-linearity is of quadratic type in the gradient of the unknown. Under suitable conditions on the parameters we prove local existence and uniqueness of a mild solution to the PDE, and investigate properties like continuity with respect to the initial condition. To conclude we consider an application of the PDE to stochastic analysis, in particular to a class of non-linear backward stochastic differential equations with distributional drivers.

Jean-Francois Jabir Higher School of Economics, Moscow

Propagation of chaos and weak constraint problems issued from Lagrangian stochastic models for turbulent flows.

Lagrangian stochastic models for turbulent models refer to a particular family of singular stochastic differential equations, nonlinear in the sense of McKean, introduced in the framework of computational fluid dynamics for describing the characteristic motion of a fluid particle. Although largely used for industrial applications, these models exhibit a large variety of original and complex mathematical problems connecting stochastic analysis, partial differential equations, kinetic models and mass transportation.

After a short presentation of the practical and specific theoretical problems related to Lagrangian stochastic models for turbulent flows, we will discuss, in a broad way, two different questions issued from these models: the question of the propagation of chaos property for singular McKean SDEs and the construction of diffusion processes endowing weak constraints (namely diffusion processes whose time-marginal distributions are restrained to lie in some sub-domain of a Wasserstein *space).*

Benjamin Jourdain, Ecole des Ponts ParisTech, CERMICS

Lifted and geometric differentiability of the squared quadratic Wasserstein distance

Any optimal coupling for the quadratic Wasserstein distance $\mathrm{W}_2^2(\mu,\nu)$ between two probability measures μ and ν with finite second order moments on d is the composition of a martingale coupling with an optimal transport map T. We check the existence of optimal couplings in which this map gives the unique optimal coupling between μ and $\mathrm{T}\#\mu$. Next, we prove that $\sigma \mapsto W_2^2(\sigma,\nu)$ is differentiable at μ in both Lions and the geometric senses if there is a unique optimal coupling between μ and ν and this coupling is given by a map.

Mark Kelbert, Higher School of Economics, Moscow

Context Dependent Information Measures: basic properties and applications

The concept of weighted entropy takes into account values of different outcomes, i.e., makes entropy context-dependent, through the weight function. We develop a novel methodology in clinical trial based on maximization of context-dependent entropy gains. Similar approach is very effective in the analysis of the so-called Kelly investments in financial markets.

On theoretical side, we analyse an analogue of the entropy-power inequality for the weighted entropy and discuss connections with weighted Lieb's splitting inequality.

Valentin Konakov, Higher School of Economic, Moscow
A Local Limit Theorem for a Robbins-Monroe Procedure
The Robbins - Monroe algorithm is a recursive, simulation-based stochastic procedure to approximate the zeros of a function that can be written as an expectation. It is known that under some technical assumptions, a Gaussian convergence can be established for the procedure. Here, we are interested in the local limit theorem, that is, quantifying this convergence on the density of the involved objects. The analysis relies on a parametrix technique for Markov chains converging to diffusions, where the drift is unbounded.

Anna Kozhina, Higher School of Economic, Moscow
Parametrix method for SDEs. Weak error estimation
We study the weak error associated with the Euler scheme of non-degenerate and Kolmogorov like degenerate diffusion processes with non-smooth coefficients. Precisely, we consider the case Hölder continuous homogeneous coefficients. With the parametrix method we estimate bounds not only for corresponding weak errors but also for transition densities differences.

Mauro Mariani, Higher School of Economic, Moscow
On the variational convergence of a class of discounted control problems
I will discuss the variational limit of a class of cost functionals associated with the empirical measure and current of deterministic control problems. Various limits are considered (e.g. long-time and vanishing discount factor), and a Gamma-convergence principle is established. As a by-product, we gather a selection principle for the solutions to Hamilton-Jacobi equations.

Mario Maurelli, University of York and University of Edinburgh
2D Euler equations with transport noise: bounded and measure-valued vorticity
We consider Euler equations in two dimensions perturbed by a transport noise (on the vorticity). We show well-posedness among bounded vorticity solutions and existence among nonnegative H^{-1} vorticity solutions, thus generalizing to the stochastic case classical results by Yudovich and Delort. Based on joint works with Zdzislaw Brzezniak and Franco Flandoli.

William Salked Univeristy of Edinburgh
Differentiability of SDEs with drift of super-linear growth
We close an unexpected gap in the literature of stochastic differential equations (SDEs) with drifts of super linear growth (and random coefficients), namely, we prove Malliavin and Parametric Differentiability of such SDEs. The former is shown by proving Ray Absolute Continuity and Stochastic Gâteaux Differentiability.

Denis Talay, INRIA
On statistical, analytical and numerical issues related to Wasserstein distances and singular McKean-Vlasov interactions
The first part of the talk will concern a calibration issue.In a joint work with Jocelyne Bion-Nadal (CMAP, Ecole Polytechnique) we consider the set of the solutions to stochastic differential equations with smooth coefficients. We introduce a Wasserstein type distance which can be represented by means of a stochastic control problem. We discuss theoretical and numerical aspects of this result

We then present the non-standard and singular McKean-Vlasov dynamics that we jointly introduced and studied with Milica Tomasevic (Inria) to analyse Keller-Segel parabolic-parabolic system. We also summarise the results obtained with Jean-Francois Jabir (HSE, Moscow) on the related particle system. We finally present our joint work with Hector Olivero (University of Valparaiso, Chile) we construct and analyse an hypothesis test which helps to detect when the probability distribution of

complex stochastic simulations has an heavy tail and thus possibly an infinite variance. This issue is notably important when simulating particle systems with complex and singular McKean-Vlasov interaction kernels. We will explain why the standard limit theorems do not lead to effective tests. Even in the simple case of iid sequences our procedure and its convergence analysis are based on deep tools coming from the statistics of semimartingales.

Alvin Tse University of Edinburgh

Weak particle expansions of McKean-Vlasov SDEs via Wasserstein calculus

We consider a stochastic process X, described by a mean-field stochastic differential equation, whose coefficients depend on the evolving law of the process itself. Such equations arise as a limit of the system of stochastic interacting particle systems $Y^{i,N}$ i.e SDEs that are coupled via the empirical law. In this talk, we show that under suitable regularity assumptions the weak error between X and $Y^{i,N}$ can be expressed as $\sum_{j=1}^{k-1} \frac{C_j}{N^j} + O(\frac{1}{N^k})$, for some constants $C_1, \ldots, C_{k-1}$ that do not depend on N. That is we formulate the weak-error particle expansion in the spirit of Talay and Tubaro. The expansion we propose relies on the powerful machinery of differentiation with respect to a probability measure, which was proposed by P. Lions in his lectures in College de France. At the core of our proof lies the study of the regularity of the PDE on measure spaces which might be of independent interest.

Michele Ottobre, Heriot-Watt University

On a class of SDEs with multiple invariant measures

In 1968 Hoermander introduced a sufficient condition to ensure hypoellipticity of second order partial differential operators. As is well known, this seminal work of Hormander had deep repercussions both in the analysis of PDEs and in probability theory and a large strand of literature has been devoted to studying ergodic properties of processes which do satisfy the Hoermander condition (HC). While such literature has mostly been concerned with study of convergence to equilibrium for dynamics which admit a unique invariant measure, it is a known fact that Hoermander-type diffusions need not be ergodic, i.e. they need not admit a unique invariant measure. In this talk we will present the UFG condition, which is weaker than the Hormander condition. Such a condition was introduced by Kusuoka and Strook with probabilistic motivations, and, independently, by Sussman, Hermann and Lobry, this time in the field of control theory. We will present new results on the geometry and long-time behaviour of diffusion semigroups that do not satisfy the Hoermander condition. We will highlight how, loosely speaking, UFG diffusions constitute a large class of SDEs which exhibit multiple equilibria (invariant measures) and such that it is possible to determine in a systematic way the basin of attraction of each equilibrium state.

Michael Roeckner, Universität Bielefeld

Nonlinear Fokker-Planck-Kolmogorov equations and stochastic distribution dependent SDE

By Ito's formula the time marginals of a solution to a distribution dependent SDE solve a nonlinear Fokker-Planck-Kolmogorov equation. This talk is about the converse: we present a general technique how to identify a solution to a nonlinear Fokker-Planck-Kolmogorov equation consisting of probability densities as the time marginals of a solution to a distribution dependent SDE. We apply this to the special case of a porous media equation perturbed by the divergence of a vector field depending nonlinearly on the solution. More precisely, we construct a generalized entropic solution u to this equation and apply the above general technique to find the corresponding distribution dependent SDE which has a weak solution with marginals given by u. We thus gain a probabilistic representation of u. The final aim is to develop a general theory relating distribution dependent SDE and non-linear Fokker-Planck-Kolmogorov equations analogous to the classical linear case.
This is joint work with Viorel Barbu (Romanian Academy, Iasi)

Xiling Zhang, University of Edinburgh

A central-limit approximation for the small jumps of multi-dimensional Lévy processes.

The small jumps cannot be ignored when approximating Lévy-SDEs, even with some relatively nice Lévy measures. To tackle this problem, Fournier borrowed a CLT result in the Vaserstein distance by Rio to approximate the small jumps with a suitable normal random variable, but both results fail in multi-dimensions. In this talk I will show that, by considering some polynomial perturbations of the normal distribution, one can generalise both Rio and Fournier's results to the multi-dimensional case, with potentially arbitrarily fast convergence rates.